Constructing Calculus Concepts through Worksheet Based Problem-Based Learning Assisted by GeoGebra Software

Yerizon [1*], I Made Arnawa [2], Nisa Fitriani [1], Nor'ain Mohd Tajudin [3]

[1] Department of Mathematics, Universitas Negeri Padang, Padang, Indonesia.

[2] Department of Mathematics, Andalas University, Padang, Indonesia.

[3] Department of Mathematics, Sultan Idris Education University, Tanjung Malin, Malaysia.

Abstract

This study aims to produce a valid, practical, and effective calculus learning worksheet using a problem-based learning method assisted by GeoGebra software in improving problem-solving skills. This worksheet serves as a guide to students in constructing the calculus concepts through the provided instructions according to the Problem-Based Learning syntax. Furthermore, the construction process was carried out using GeoGebra software. Students are given scaffolding to construct concepts. This research is development research. This study employs a Plomp model, which consists of three stages, namely the preliminary study, development or prototyping, and assessment phases. The subjects are 32 students in class XI.2 at the State Senior High School of 14 Padang. The results showed that the worksheet produced was valid, practical, and effective in improving students' problem-solving abilities. This implies that it would assist students in constructing calculus concepts. Furthermore, it showed that GeoGebra is capable of visualizing abstract calculus concepts, which enables students to understand higher mathematical thinking on the main concept of calculus. In conclusion, the use of GeoGebra in mathematics learning is able to help students construct mathematical concepts and improve their problem-solving abilities.

Keywords: Calculus; GeoGebra; Problem-Based Learning; Plomp Model.

1. Introduction

Mathematics is a very abstract subject for students, which makes it difficult for teachers to carry out the learning process in class [1]. Calculus is very important in modern science and technology [2]. However, many studies have reported that students find it difficult to understand [3, 4]. Difficulties occur because they have limited prior knowledge of learning calculus [5]. Some topics which are quite difficult for students to understand include limits [6, 7]. Students have difficulty understanding the relationship between the graphs of a function in the form of a parameter with derivatives [8-10]. They also have difficulty determining the definition of a definite integral [11, 12]. The low knowledge of students on the topic of calculus also has an impact on other subjects [13, 14].

The difficulties faced by students are mainly in the visualization of abstract concepts, and the alternative solution to overcome the difficulties is to integrate the use of technology into the process of teaching and learning. The use of technology can be beneficial as it enhances problem-solving, critical and creative thinking, as well as mathematical thinking skills [1]. Currently, there are many types of software for learning mathematics that are capable of improving

students' understanding [15] and helping in eliminating misconceptions [16]. Furthermore, they are capable of carrying out calculations, painting graphics, creating tables, and creating various representations [17].

Meanwhile, various attempts have been made to overcome these difficulties. Case & Speer [18] examined strategies that may be used by teachers to assist students in understanding the abstract ideas of calculus and reasoning in the calculus theorem. Furthermore, Adams & Dove [4] used flipped learning to improve students' learning outcomes and perceptions of calculus. Carnell et al. [5] examined the difference in the ability of students that have taken calculus and those that have yet to. Dawkins and Epperson examined traditional calculus learning with problem-solving-oriented labs. However, these studies did not yield good results because the calculus material is still abstract for students [3]. Therefore, it is necessary to involve students in constructing the calculus concept. This is because the concept is abstract and needs to be visualized for easy understanding. By visualizing, it becomes possible to improve the calculus learning process by involving students in problem-solving, modeling concepts, and solving open-ended problems [19]. Consequently, they are able to understand the calculus concept comprehensively [20].

One software that is used and a free, open-source mathematics software program, capable of carrying out various representations and mathematical explorations that are easy to use by teachers and students is GeoGebra. This software is able to overcome the difficulties experienced by students [1]. Understanding mathematical ideas and concepts would be easier through various representations and their relationships. GeoGebra is very supportive of drawing points, lines and all conics. Furthermore, it has a special feature capable of finding the important points of a function, for example, the extreme points of a function. It also has the ability to deal with numerical, vector, point, find derivatives and carry out integral functions as well as offer commands such as roots or extreme values [21].

Using the GeoGebra program, it becomes possible to visualize and manipulate abstract geometry objects quickly, accurately, and efficiently. This program functions as a learning medium that provides visual experiences to students in interacting with geometry concepts [22].

GeoGebra is very useful as a medium for learning mathematics with a variety of activities, which include [23-25]:

- As a demonstration and visualization medium for certain mathematical concepts. Therefore, it produces geometric drawings quickly and with greater precision than using a pencil, ruler or compass. Furthermore, it is used to ensure that the painting is correct.

- As a construction tool for certain mathematical concepts with animation and manipulation movement (dragging) facilities, which aids in providing clearer visual experience to students in understanding geometric concepts.

- As a tool for the discovery process to find a mathematical concept and the correct answer to mathematical calculations. Therefore, making it easier for teachers or students to investigate the properties that apply to a geometric object.

Many studies have reported that GeoGebra mathematical software is capable of improving the mathematics learning process by involving problem-solving, modeling mathematical concepts and solving open-ended problems. Therefore, it enables students to understand mathematical concepts in a comprehensive manner [19, 20].

Tatar and Zengin [1] discovered that GeoGebra is an effective tool for teaching and learning mathematics. Furthermore, it is effective in improving students' proof of ability and used as a medium to help users construct mathematical concepts. It also demonstrates or visualizes mathematical concepts [26, 27]. Farihah [28] reported that the learning outcomes of students using GeoGebra in learning straight line equation graphs were better than those that did not. The process of students building their own knowledge, with its assistance, generates new and more meaningful knowledge. Khalil et al. [29] and Alkhateeb and Al-Duwairi [30] also reported that GeoGebra has a significant influence on students' mathematical abilities. Therefore, teachers should design mathematics learning with the help of GeoGebra because students understand better when they see the visual form of a math topic.

Previous research using GeoGebra has been carried out by several researchers. Tatar and Zengin [1] use GeoGebra in constructing the concept of definite integral. Maravillas et al. [7] used GeoGebra to improve students' understanding of the concept of limit. Bulut et al. [20] investigated the use of GeoGebra in constructing the concept of division. Kusumah et al. [22] looked at the effect of using GeoGebra in learning three-dimensional geometry. Joshi & Singh [26] discussed the use of GeoGebra in understanding the concept of linear equations. Based on this, it is necessary to study the use of calculus concepts, especially the relation of derivatives to the slope of the tangent line.

Furthermore, it is necessary to make worksheets that would assist students in constructing calculus concepts with the help of GeoGebra. In the construction process, problem-based learning stages were used, which includes several phases of learning, namely 1) Orientation Phase, which involves exposing students to real-world problems, 2) Engagement phase, where students are involved in problem-solving activities, 3) Inquiry and Investigation phase, where students carryout investigations to solve problems and 4) Debriefing, where students carry out questions and answers as well as discussions related to problem-solving activities [31].

In this worksheet, instructions on how to use GeoGebra were provided in order to assist the students in constructing calculus concepts. Therefore, this study aims to produce valid, practical and effective student worksheets based on problem-based learning assisted by GeoGebra (LKPD-PBL-G). The worksheet should be valid in terms of content, presentation, language, graphics, and practical in terms of implementation, time, ease of use, as well as effective in terms of its potential impact on solving students' mathematical problems. The questions that need to be answered in this study are:

- How is the validity of LKPD-PBL-G based on the aspects of content, presentation, language, and graphics?

- How is the practicality of LKPD-PBL-G based on the aspects of implementation, time and ease of use?

- How is the effectiveness of LKPD-PBL-G based on its potential impact on students' problem-solving abilities?

2. Methodology

This study employs a Plomp model of development, which was developed by *Tjeerd Plomp*. It consists of three stages, namely preliminary research [32], development or prototyping phase, and assessment phase [33]. Furthermore, this model has been used by many researchers such as [34, 35].

2.1. Preliminary Research Phase

This stage aims to ascertain what is needed for the development of the learning tools that will be produced. It is divided into several activities, which include needs, student, curriculum, and concept analysis. Needs analysis was carried out by interviewing teachers, observing learning in class and providing questionnaires as well as test questions to ascertain students' mathematical problem-solving abilities. Student analysis was carried out to determine the characteristics of students, which include academic abilities, hobbies, preferred worksheet forms and preferred colors. Curriculum analysis was carried out to determine the indicators, sequence and coverage of the material required according to the predetermined core competencies. Lastly, concept analysis was carried out by identifying the main concepts, detailing, and systematically arranging the teaching material according to the presentation order.

2.2. Development or Prototyping Phase

Based on the preliminary analysis, a worksheet was developed. Subsequently, a formative evaluation was carried out by taking into account the feasibility of content, language, presentation and graphics. The formative evaluation was carried out in accordance with the steps suggested by Tessmer (1993) [32] as shown in Figure 1.
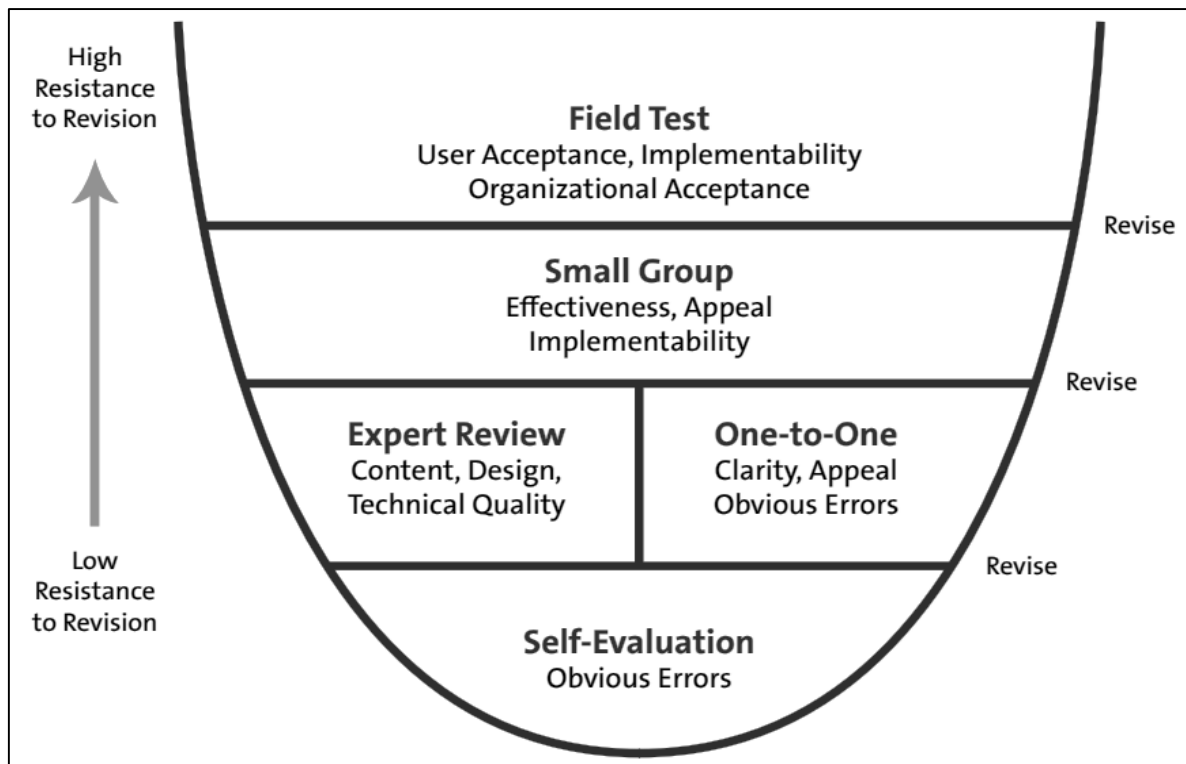

Figure 1. Formative evaluation of Tessmer's (1993) development

Self-evaluation involves the assessment of products that have been designed using a checklist of product characteristics or specifications (the result is called prototype 1). Expert review involves seeking the opinion of an expert i.e. asking for expert opinion to provide assessments and suggestions on products, in order to determine the validity in regards to feasibility of content, language, presentation and graphics (the result is prototype 2). The average validation result data from experts were categorized into five as shown in Table 1 [36].

Table 1. Average validation score criteria

Level of achievement	Qualification	Information
90% - 100%	Very good	No need to revise
75% - 89%	Good	Revised as necessary
65% - 74%	Enough	Pretty much revised
55% - 64%	Less	Much revised
0 - 54%	Very less	Revised in total

2.2.1. One-to-one Evaluation Activity

In this activity, the students were called one by one using a worksheet. The work process observed was to be used by students, recording their comments and asking about the difficulties they are experiencing (the result is prototype 3). The details of the assessment aspects are shown in Table 2.

Table 2. One-to-one evaluation aspects

No.	Assessment aspect	Assessment component
1	Content	Difficulty level, clarity, activity instructions, attractiveness, recency of material.
2	Learning Design	Readability, clarity of learning objectives, logical logic of material delivery
3	Implementation	The level of ease and / or difficulty of use, possible difficulties encountered, and others.

2.2.2. Small Group Evaluation

Small group evaluation was carried out using a small group worksheet, which consisted of 6 students with heterogeneous abilities. It aims to identify deficiencies in the prototype 3 worksheets from the aspects of presentation, readability, implementation, suitability of time allocation, and ease of learning tools use. The instrument used were the implementation observation sheet and participant's response questionnaire (the result is called prototype 4).

Data collection was carried out through interviews and distribution of questionnaires. The questionnaires contained questions posed to students and teachers, which include about the presentation of LKPD-PBL-G, aspects of ease of use, time and readability. The criteria for the questionnaire are shown in Table 3 [37].

Table 3. Categories of practicality questionnaire results on LKPD-PBL-G (student and teacher responses)

No.	Level of achievement (%)	Category
1	81 – 100	Very Practical
2	61 – 80	Practical
3	41 – 60	Quite Practical
4	21 – 40	Less Practical
5	0 – 20	Impractical

2.3. Assessment Phase

Based on the results of the small group, the worksheet was revised (called prototype 4). Furthermore, prototype 4 was tested again at the field test stage. The aim was to determine the effectiveness of using LKPD-PBL-G to increase students' mathematical problem solving abilities. This worksheet was examined using two groups of students with equal abilities in order to ascertain whether both groups would produce positive results on the use of LKPD-PBL-G. The criteria used were 75 in accordance with the minimum completeness criteria set by the school and the t-test was used. In general, the development process can be seen in Figure 2.
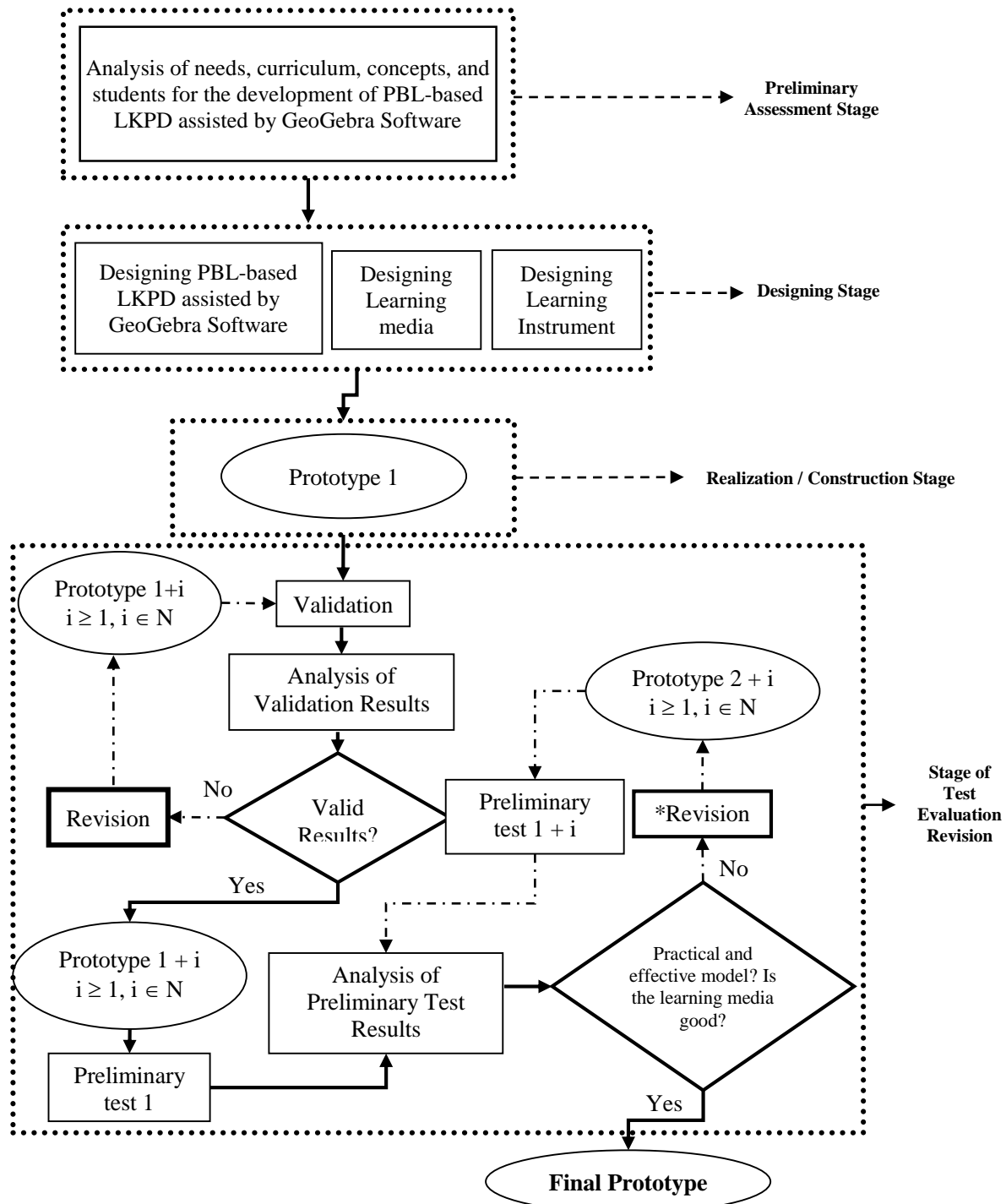
Figure 2. LKPD-PBL-G Development Procedure

3. Results and Discussion

3.1. Preliminary Analysis

From the needs analysis activity, it was seen that students' interest in learning mathematics was still lacking. This is because, they think that mathematics is a difficult and less important subject to learn, therefore they do not put in serious efforts. Only a small proportion of students understood and took the lesson seriously. Furthermore, the students rarely asked questions during the learning process and only accept what the teacher says. Through this process, it is unlikely that their mathematical ability would developed, especially problem-solving ability. This is evident from the large number of students that were unable to solve non-routine story questions. Furthermore, they were unable to present a problem formulation systematically in various forms and only imitate the examples of solving the problems given. This is thought to have occurred due to the inability of the learning tools (especially Student Worksheets) developed by the mathematics teacher to increase students' motivation and interest in learning mathematics.

Therefore, analysis of students' characteristics is needed before designing Worksheets (LKPD). This is required to ascertain which characteristics should be used as instructions in learning planning. Based on observations in class XI students, the results obtained include:

- Students have high curiosity.

- Students are less focused and have difficulty concentrating on learning.

- Students prefer to study in groups.

- Students prefer a worksheet that is attractive, easy to understand and colourful. The preferred colour was green, while the desired size of the student worksheet was commonly used and the most preferred font was Comic Sans MS.

3.2. Development or Prototyping Results

Characteristics of Student Worksheets Based on Problem Based Learning (PBL) assisted by GeoGebra (LKPD-PBL-G). The characteristics seen through the LKPD-PBL-G cover were in the form of title, subject matter title, pictures related to learning material that support the cover display and owner identity in the form of name, class and school. Furthermore, education unit, semester and name of the researcher were included. In addition, LKPD-PBL-G was equipped with a foreword from the researcher, which contained gratitude and the researcher's hopes. The writing of the foreword was carries out with Berlin Sans FB font in size 12. Meanwhile, learning objectives and student learning instructions were made in Comic Sans MS font with a size of 12.

The activities in the LKPD-PBL-G followed the PBL learning stages. The first step in this PBL model was student orientation to the problem. One example of a topic on the LKPD-PBL-G is the slope of the tangent to a curve at a certain point. The students were given the problem in the following Figure 3.
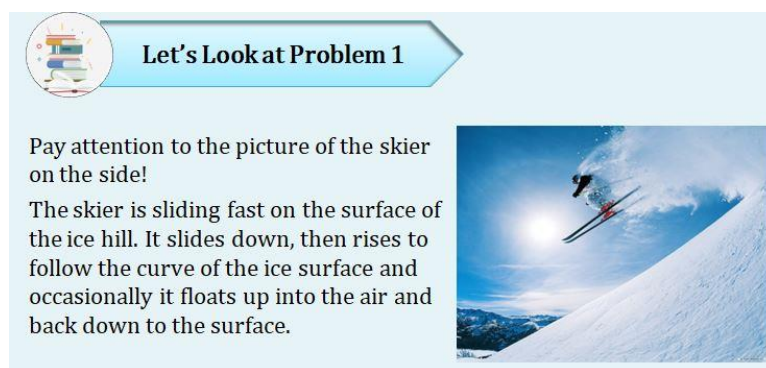

Let's Look at Problem 1

Pay attention to the picture of the skier on the side!

The skier is sliding fast on the surface of the ice hill. It slides down, then rises to follow the curve of the ice surface and occasionally it floats up into the air and back down to the surface.

Figure 3. Example of problem orientation in LKPD-PBL-G

This problem is one use of the derivative in determining the slope of a curve and instantaneous velocity. Students would be able to determine the speed of the skier at a point by finding the slope of the line at that point. Furthermore, they were asked to sit in accordance with their groups to complete activities at the LKPD-PBL-G in order to construct the concept of relating the slope of the tangent to the derivative. The activities that were carried out at LKPD-PBL-G are shown in Figure 4.
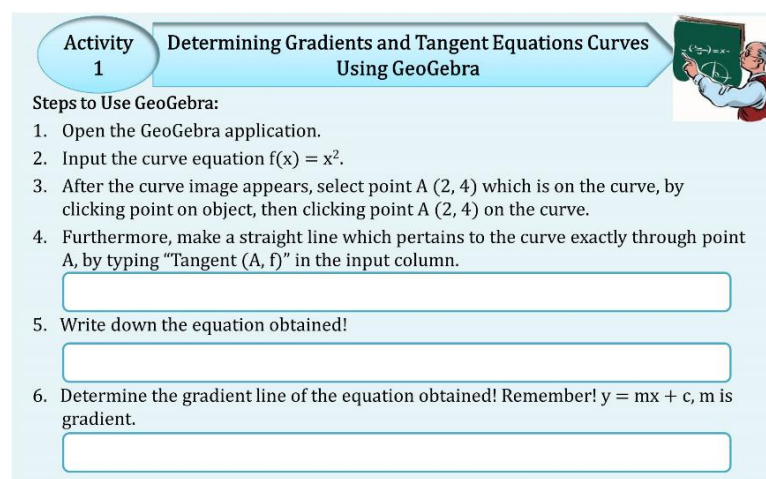

Activity 1 — Determining Gradients and Tangent Equations Curves Using GeoGebra

Steps to Use GeoGebra:
1. Open the GeoGebra application.
2. Input the curve equation $f(x) = x^2$.
3. After the curve image appears, select point A (2, 4) which is on the curve, by clicking point on object, then clicking point A (2, 4) on the curve.
4. Furthermore, make a straight line which pertains to the curve exactly through point A, by typing "Tangent (A, f)" in the input column.
5. Write down the equation obtained!
6. Determine the gradient line of the equation obtained! Remember! $y = mx + c$, m is gradient.

Figure 4. Instructions for drawing tangents with GeoGebra

At this time, the teacher's job was to act as a facilitator, i.e. guiding individual and group investigations to solve these problems. Furthermore, the students were assisted in finding a solution.

Firstly, they were instructed to paint a $y = x^2$ curve using GeoGebra. Furthermore, they were instructed to choose any point on the curve to determine the equation of the tangent (in this case the point (1, 1) was selected) with GeoGebra. The equation of the tangent was $y = 2x - 1$ with gradient 2. The result is shown in Figure 5.
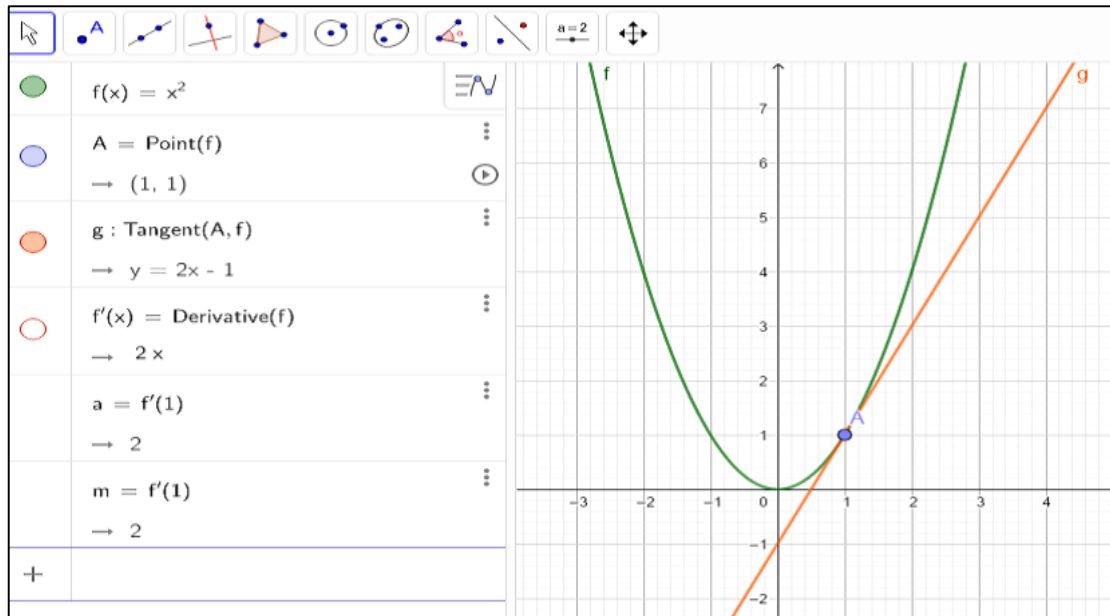

Figure 5. Graph of function $y = x^2$ and tangent at point (1, 1)

The students were also reminded about the gradient of a straight line $y = mx + c$, where m is the gradient. Therefore, they were aware that $m = 2$.

After the students produced the picture, they were guided to understand the concept of finding the slope of the tangent by using derivatives. The steps that were carried out are shown in Figure 6.

Figure 6. Steps to find the slope of the tangent using the derivative

First, the students were asked to find the value of the first derivative of the function $f(x) = x^2$ at point (1, 1). They obtained $f'(x) = 2x$, by substituting $x = 1$, therefore $f'(1) = 2(1) = 2$ was obtained. Furthermore, they were asked to ascertain the relationship of the tangent gradient obtained by GeoGebra with the derivative of the function f in (1, 1). Based on this activity, the students concluded that the slope of the tangent to the curve is the same as the first derivative of the curve function at a point or $m = f'(x)$.

3.2.1. LKPD-PBL-G Validation

The learning device designed based on the results of preliminary study was called prototype I. The results of prototype I were validated through 2 stages, namely self-evaluation and expert review. The two stages are explained as follows.

Self-Evaluation Results

In the self-evaluation stage, the prototype I was examined again with the help of colleagues from the same department. It was examined for errors in typing letters, the use of punctuation marks in sentences, clarity of the images used, and suitability of the material in LKPD-PBL-G, suitability of the pictures to the problem and sequence of activities. Several types of typos, punctuation, and image layout errors were discovered. After the corrections were made, the LKPD became prototype II.

Expert Validation

The LKPD-PBL-G was validated by 5 experts. The results of the validation are shown in Table 4.

Table 4. Results of LKPD-PBL-G validation by experts

No.	Rated aspect	Validity Index	Category
1	Didactic	89 %	Good
2	Content eligibility	86 %	Good
3	Language	94 %	Very Good
4	Graphics / Layout	81 %	Good
	Average	**88 %**	**Good**

Several suggestions were made by the validator for this LKPD-PBL-G, for example separating activities using GeoGebra from the content without GeoGebra. After the validation process through self-evaluation and expert review was completed, improvements were made to prototype 1 according to the validator's suggestions, the results were called prototype 2. Furthermore, a practicality test was carried out on prototype 2.

3.2.2. Practical LKPD-PBL-G

The practicality test of prototype 2 was carried out in 2 stages, namely one to one evaluation and a small group evaluation.

3.2.2.1. One-to-one Evaluation Results

The one-to-one activity was carried out by 3 students in class XI.2 at State Senior High School of 14 Padang, that were selected by math teachers of class XI based on their high, medium, and low ability levels. In this trial, they were instructed to sit separately and try to fill in the LKPD-PBL-G according to their respective abilities. Furthermore, they were instructed to comment on the given LKPD-PBL-G.

One to one evaluation aims to observe instructions, record responses, suggestions, as well as sentences that were difficult to understand from LKPD-PBL-G and at the end of this activity, the practicality of using the Students Worksheet was seen. This individual evaluation was carried out 6 times at 6 LKPD-PBL-G with the main subject being the application of algebraic function derivatives.

The work of LKPD-PBL-G started with student activities to observe and assess LKPD-PBL-G from the cover page and instructions for use. The result showed that the three students liked the cover page and understood the instructions for using LKPD-PBL-G well. Meanwhile, when working on LKPD-PBL-G 1, namely regarding gradients, tangent equations and normal line equations of a curve, student 2 was constrained in making a curve tangent. He was unable to use the "tangent" icon and did not click on point A followed by a $f(x)$ curve. Therefore, the image did not appear. The following was the dialogue between the researcher and students 2.

Researcher: "Student, do you have any doubts you want to ask?"

Student 2: This ma'am, I have followed the steps in LKPD-PBL-G. However, the curve is yet to appear.

Researcher: "Try again, while I watch, is the method you used correct or not?"

Student 2: "Yes ma'am"

Researcher: "Now, after you select the tangent icon, try clicking point A and then clicking the curve that was created earlier"

Student 2: (Starts practicing)

Researcher: "See, you can do it?"

Student 2: "O yes, I can ma'am".

After the dialogue with student 2, the researcher asked the three students to observe the laptop screen and instructed them to write in the LKPD-PBL-G about the curve tangent equation that had appeared in the GeoGebra application. Student 1 mentioned that the curve tangent equation was $y = 2x - 1$. Subsequently, students 2 and 3 confirm what student 1 said. The following was the conversation between students and the researcher.

Researcher: "You are all right, now try to pay attention to the equations that have been found using the GeoGebra application. Is it possible to determine the slope of the tangent to the curve? Now, you were previously told that, " $y = mx + c$". still remember?"

Student 1: "Yes, ma'am. Therefore, the gradient is 2 right. In the equation $y = 2x - 1$, that means $m = 2.$"

Researcher: "Yes, that's right. Does everyone understand? By using the GeoGebra application, it is possible to view the mage immediately and find out the equations and gradients of the curves as well. Now let's continue activity 2. "Student: "OK ma'am".

The students started their next activity, namely problem solving questions at LKPD-PBL-G. The problem given was about the curve tangent equation and the normal line, as well as determining the gradient of the equation. The results obtained are shown in Table 5.

Table 5. Scores of problem-solving tests in one-to-one evaluation

Indicator	Students Score		
	Student 1	**Student 2**	**Student 3**
1	4	4	4
2	4	4	3
3	3	3	3
4	3	2	2
5	3	2	2
Total	17	15	14
Percentage	85	75	70

Based on the answers provided, it was seen that the students were able to organize data and select relevant information in identifying problems. Furthermore, they were able to present the problem formulation mathematically on the questions, but have not been able to correctly choose and use the right approaches and strategies to solve them. The results of the interview on the one to one evaluation activity are shown in Table 6.

Table 6. The results of the interview on the one-to-one evaluation activity

No.	Assessment aspect	High student	Moderate student	Low student
1	Presentation	The instructions were clear, complete and easy to understand.	The instructions were clear, complete and understandable.	The instructions were quite clear and complete.
		The paper size made it easy to be used.	The paper size made it easy to be used.	The paper size made it easy to be used.
		Image position was correct.	Image position was correct.	Image position was correct.
2	Ease of use	Easy to use.	Quite easy to use.	Quite easy to use.
		The illustrations used were clear.	The illustrations used were quite clear.	The illustrations used were quite clear.
3	Time allocation and readability	The time provided was sufficient.	The time provided was less.	The time provided was less
		Instructions were clear and understandable.	Instructions were clear and understandable.	Instructions were clear and understandable.

3.2.2.2. Small Group

Small group evaluation was carried out on 6 students consisting of 2 each with high, moderate and low ability. The results of observations on the practicality of learning carried out by the teacher are shown in Table 7.

Table 7. Analysis results of the LKPD-PBL-G implementation observation sheet

Meeting	I	II	III	IV	V	VI	Average
Observation value (%)	97.2	94	88	84	84	85.7	88.8
Criteria	Very practical	Very practical	Very practical	Practical	Very practical	Very practical	Very practical

The overall practicality value of implementation was 88.8% in the very practical category. Although, it did not always increase, the observation results for each meeting were still in practical criteria.

In the first stage, students were asked to observe and understand problem 1 in LKPD-PBL-G. Furthermore, they were instructed to determine the gradient and the equation of the tangent to the curve using GeoGebra. At the beginning of the first meeting, there were several notes from the researchers on the learning process carried out. Furthermore, the students were not very enthusiastic about it and both groups had difficulty using the GeoGebra application because it was the first time. However, they still tried to follow the steps in activity 1 until both groups became interested in using the application. Both groups stated that it was easier to make graphs using the GeoGebra app and obtaining the equation of the desired curve or line. The two groups continued the activity of determining the gradient of the tangent to the curve using the first derivative of the curve function at point A (2, 4).They discovered that the gradient of the tangent to a curve equals the first derivative of the curve function at the point of contact.

After the learning activities at meeting 6 were completed, the researcher asked the students for their opinions to find out the practicality of the LKPD-PBL-G used. They were asked about several aspects, namely the presentation of the LKPD-PBL-G, ease of use, time and readability. Furthermore, they were asked to fill out a questionnaire about the LKPD-PBL-G that was carried out. The questions posed to students were about the presentation of the LKPD-PBL-G, ease of use, time and readability. The results are shown in Table 8.

Table 8. Recapitulation of average LKPD-PBL-G practicality questionnaire results (student response)

No.	Statement items	Average	Percentage	Criteria
	Presentation Aspects			
1	Instructions are easy to understand.	3.5	87.5	Very practical
2	The size and font used are interesting.	3.67	91.67	Very practical
3	The problems presented are interesting and challenging.	3.5	87.5	Very practical
4	The colour combination used is interesting.	3.3	83.33	Very practical
	Ease of Use Aspects			
5	The problems presented are easy to understand.	3.5	87.5	Very practical
6	The instructions given are clear.	3.3	83.33	Very practical
7	The sentences and questions are easy to understand.	3.16	79.16	Practical
8	The illustrations was of assistance in understanding the problem.	3.16	79.16	Practical
9	The activities on the worksheet help develop the flow of thinking in communicating with friends.	3.5	87.5	Very practical
10	The activities on the worksheet was of assistance in understanding the subject matter collaboratively.	3.3	83.33	Very practical
11	Activities on the worksheet assisted in getting used to thinking, asking questions and discussing.	3.5	87.5	Very practical
12	Activities on the worksheet provide freedom to express opinions, therefore I am more confident.	3.16	79.16	Practical
13	The instructions for using GeoGebra are easy to understand.	3.5	87.5	Very practical
14	Using GeoGebra can help me be more active in the learning process.	3.3	83.33	Very practical
15	Using GeoGebra can make it easier for me to solve the problems given.	3.5	87.5	Very practical
	Readability Aspects			
16	I can read the font size clearly.	3.3	83.33	Practical
17	The language used is easy for me to understand.	3.5	87.5	Very practical
	Time Allocation Aspects			
18	There is sufficient time to work on the worksheet.	3.5	87.5	Very practical
	Average	3.43	85.56	Very practical

Based on Table 8, it was seen that LKPD-PBL-G shows one aspect with a practical category, namely ease of use with a value of 83.55% and three other aspects with a very practical category, namely presentation, readability and time allocation with values of 85.64%, 85% and 87.5%, respectively. Furthermore, the practical percentage of student worksheet based PBL assisted by GeoGebra was 85.56% in the very practical category.

Furthermore, at the small group practicality stage, an assessment was also requested from the teacher as the observer. The aspects that were assessed by the teacher include attractiveness, process of use, and ease of use, time and equivalence. The results of the teacher's response are shown in Table 9.

Table 9. Recapitulation of average LKPD-PBL-G practicality questionnaire results (teacher response)

No.	Statement Items	Average	Percentage	Criteria
1	Instructions for use are easy to understand.	4	100	Very practical
2	The problems given at the beginning of the lesson are appropriate to stimulate students to carry out activities.	4	100	Very practical
3	The work steps on the worksheet are easy to understand.	3	75	Practical
4	The questions and commands on the worksheet clearly guide students.	4	100	Very practical
5	The material is adapted to the thinking level of students.	3	75	Practical
6	The language used is communicative and can be understood by students.	4	100	Very practical
7	The images used in the LKPD help students understand the problems presented.	4	100	Very practical
8	The use of LKPD makes students more active in learning.	4	100	Very practical
9	The GeoGebra application is easy to be used in the learning process.	3	75	Practical
10	The GeoGebra application can help teachers and participants in the learning process and problem-solving.	4	100	Very practical
11	The time allocation is sufficient.	3	75	Practical
12	The material presented in the worksheet is according to the 2013 curriculum.	3	75	Practical
13	Worksheets can be used as a learning resource.	4	100	Very practical
14	Worksheets can be used as a variation in the use of learning resources.	3	75	Practical
Average		3.57	89.29	Very practical

Table 9 shows that two aspects were categorized as practical, namely time and equivalence. Meanwhile, three other aspects were categorized as very practical, namely attractiveness, process of use and ease of use. Furthermore, the average LKPD-PBL-G practicality value was 89.29% with the very practical category.

3.3. Assessment Phase

The effectiveness test was carried out through a problem-solving ability test on students. They were divided into two groups with heterogeneous abilities and the test was repeated thrice to ascertain the progress. The test results are shown in Table 10.

Table 10. Score of problem solving ability test

Indicator	Group 1				Group 2			
	Test 1	Test 2	Test 3	Total	Test 1	Test 2	Test 3	Total
1	4	4	4	12	4	4	4	12
2	4	4	4	12	4	4	4	12
3	4	4	3	11	3	4	4	11
4	3	4	3	10	3	3	3	9
5	3	3	3	9	3	3	3	9
Total	18	19	17	54	17	18	18	53
Percentage	90	95	85	90	85	90	90	88.3

Based on the results above, it was seen that the percentage of student mastery in each test was above 85%. This has already exceeded the student's previous level of mastery. Therefore, the LKPD-PBL-G was stated to be very effective. Furthermore, the results of the two groups were compared using a t-test (Table 11).

Table 11. The t-test results of students' problem solving abilities

	Levene's test for equality of variances		t-test for equality of means				
	F	Sig.	t	df	Sig. (2-tailed)	Mean difference	Std. error difference
Equal variances assumed	0.480	0.508	0.224	8	0.829	0.20000	0.89443
Equal variances not assumed	-	-	0.224	7.824	0.829	0.20000	0.89443

Based on the results of the t-test, it was seen that the sig. = 0.829 > 0.05. This is because, with sig. > 0.05, it can be concluded that the problem-solving abilities of the two groups are not different. This implies that LKPD-PBL-G has the same effect on both groups. Since the mastery of problem-solving in group I reached 90% and group II reached 88.3%, both groups reached the minimum criteria set. Therefore, LKPD-PBL-G is effective in improving students' problem-solving abilities.

3.4. Discussion

Based on data analysis, it was discovered that LKPD-PBL-G could be used easily by students, therefore it was possible to improve their problem-solving abilities. This implies that this worksheet is capable of effectively assisting students in learning. GeoGebra allows students to construct and animate geometric objects, which makes it easier to explore interactively [38]. Furthermore, it was possible to carry out construction and exploration from geometric shapes and graphs of an equation dynamically [39]. Learning mathematics becomes exploratory where students directly and instantaneously ascertain the relationship between analytic and visual representations of a concept, as well as the relationship between mathematical concepts. Therefore, this software is usable in making mathematical concepts dynamic [40].

The use of technology is something that needs to be carried out in the learning process, especially in mathematics. It is usable to explain abstract mathematical concepts. Furthermore, its use contributes to problem-solving, critical and creative thinking, mathematical thinking [1] and mathematical visual thinking of students [41].

From the visualization carried out, it is possible for students to conclude the geometry subject matter. The use of dynamic mathematics software saves time significantly, therefore students are able to concentrate on assignments, which are more conceptually oriented. This is one advantage of using computer media in learning, which makes it easier and faster for students to understand concepts [42].

Several other studies have also shown that GeoGebra is very effective in learning mathematics. Hidayati and Kurniati [43] discovered that learning using GeoGebra-assisted Spatial Geometry teaching materials is capable of improving students' critical thinking skills. Dwijayani [44] reported that learning using GeoGebra improves students' mathematical problem-solving abilities and stimulates students' creative thinking. Supriadi [45] stated that the use of GeoGebra in geometry courses has a positive response from students, namely 80.43%. Furthermore, GeoGebra is able to improve students' mathematical spatial abilities [46].

Students can use GeoGebra to assist in learning mathematical concepts, especially algebra and geometry. Abstract mathematical concepts can be visualized with the help of GeoGebra. Students are asked to explore the properties of a mathematical concept based on the instructions given. By doing exploration, students have new experiences that are meaningful from the activities that have been carried out. Students will observe the visualization of the given concept during exploration, so they can conclude something from the phenomenon. New ideas will emerge, when they discover something new from the GeoGebra display. GeoGebra can manipulate abstract mathematical objects into reality through various representations.

With the assistance of GeoGebra, students can also do self-assessment on the questions that have been answered. After completing the answer, they can check their answer using GeoGebra. They will know whether the work is right or wrong. This activity makes them have the certainty to continue to the next activity. Thus, they will be motivated to learn the next topic.

In addition to improving mathematical abilities, geometry learning with GeoGebra also makes students have a positive attitude towards mathematics, making them more enthusiastic about learning [46, 47]. Furthermore, geographical-assisted STAD-type cooperative learning is capable of improving students' geometric problem-solving skills and mathematical dispositions. Therefore, students' geometry learning outcomes were increased by participating in realistic mathematics education learning using GeoGebra media [48, 49].

4. Conclusion

Based on the results obtained, it was concluded that LKPD-PBL-G is effective in helping students construct mathematical concepts and improve their problem-solving abilities. The construction process with GeoGebra is carried out by providing scaffolding in the form of instructions in the LKPD-PBL-G. Students can understand the concept through the provided activities. They can observe the relationship between the tangent gradient and the concept of derivative in a meaningful way. Students can see the relationship between calculus concepts and their application to other mathematical topics. Therefore, GeoGebra is able to visualize abstract mathematical concepts, making them easy for students to understand. Furthermore, its use in mathematics learning raises students' positive attitudes towards mathematics. It is suggested that teachers use GeoGebra software in teaching mathematical concepts to students. Furthermore, they may design a worksheet to guide students in finding a concept or formula. In this way, learning becomes more meaningful to them.

5. Declarations

5.1. Author Contributions

5.2. Data Availability Statement

The data presented in this study are available in article.

5.3. Funding

5.4. Ethical Approval

Not applicable.

5.5. Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

6. References

[1] Tatar, E., & Zengin, Y. (2016). Conceptual Understanding of Definite Integral with GeoGebra. Computers in the Schools, 33(2), 120–132. doi:10.1080/07380569.2016.1177480.

[2] Çekmez, E. (2020). Establishing the link between the graph of a parametric curve and the derivatives of its component functions. International Journal of Mathematical Education in Science and Technology, 51(1), 115–130. doi:10.1080/0020739X.2019.1663950.

[3] Dawkins, P. C., & Epperson, J. A. M. (2014). The development and nature of problem-solving among first-semester calculus students. International Journal of Mathematical Education in Science and Technology, 45(6), 839–862. doi:10.1080/0020739X.2014.884645.

[4] Adams, C., & Dove, A. (2018). Calculus Students Flipped Out: The Impact of Flipped Learning on Calculus Students' Achievement and Perceptions of Learning. Primus, 28(6), 600–615. doi:10.1080/10511970.2017.1332701.

[5] Carnell, L., Graham-Squire, A., O'Hara, K., & Piechnik, L. (2018). Environmental Impacts: How Comparative Prior Knowledge Affects Students' Calculus Experience. Primus, 28(10), 920–935. doi:10.1080/10511970.2018.1472682.

[6] Oktaviyanthi, R., Herman, T., & Dahlan, J. A. (2018). How does pre-service mathematics teacher prove the limit of a function by formal definition? Journal on Mathematics Education, 9(2), 195–212. doi:10.22342/jme.9.2.5684.195-212.

[7] Aréchiga Maravillas, J. C., Díaz, S. A., Salazar-Torres, J., & Andrade, D. A. S. (2019). Testing GeoGebra as an effective tool to improve the understanding of the concept of limit on engineering students. Journal of Physics: Conference Series, 1408(1), 012008. doi:10.1088/1742-6596/1408/1/012008.

[8] Nuphanudin, N., Komariah, A., Shvetsova, T., Gardanova, Z., Podzorova, M., Kurniady, D. A., ... & Kosov, M. (2022). Effectiveness of Students' Motivation Factors in the Competency-Based Approach: A Case Study of Universities in Russia and Indonesia. Emerging Science Journal, 6(3), 578-602. doi:10.28991/ESJ-2022-06-03-012.

[9] Ocal, M. F. (2017). The Effect of Geogebra on Students' Conceptual and Procedural Knowledge: The Case of Applications of Derivative. Higher Education Studies, 7(2), 67. doi:10.5539/hes.v7n2p67.

[10] Widada, W., Herawaty, D., Andriyani, D. S., Marantika, R., Yanti, I. D., & Falaq Dwi Anggoro, A. (2020). The thinking process of students in understanding the concept of graphs during ethnomathematics learning. Journal of Physics: Conference Series, 1470(1), 1–6. doi:10.1088/1742-6596/1470/1/012072.

[11] Serhan, D. (2015). Students' understanding of the definite integral concept. International Journal of Research in Education and Science, 1(1), 84–88. doi:10.21890/ijres.00515.

[12] Santos, E. M. (2019). A look into students' conceptual understanding of the definite integral via the APOS model. The 2nd international conference on science, mathematics, environment, and education. AIP Conference Proceedings, 2194(1), 020110. doi:10.1063/1.5139842.

[13] Hoban, R. A. (2019). A resource for introducing students to the integral concept. International Journal of Mathematical Education in Science and Technology, 50(4), 603–616. doi:10.1080/0020739X.2018.1480809.

[14] Fatimah, S., & Yerizon. (2019). Analysis of difficulty learning calculus subject for mathematical education students. International Journal of Scientific and Technology Research, 8(3), 80–84.

[15] Pilli, O., & Aksu, M. (2013). The effects of computer-assisted instruction on the achievement, attitudes and retention of fourth grade mathematics students in North Cyprus. Computers and Education, 62(1), 62–71. doi:10.1016/j.compedu.2012.10.010.

[16] Gürbüz, R., & Birgin, O. (2012). The effect of computer-assisted teaching on remedying misconceptions: The case of the subject "probability." Computers and Education, 58(3), 931–941. doi:10.1016/j.compedu.2011.11.005.

[17] Aristika, A., & Juandi, D. (2021). The effectiveness of hybrid learning in improving of teacher-student relationship in terms of learning motivation. Emerging Science Journal, 5(4), 443-456. doi:10.28991/esj-2021-01288.

[18] Case, J., & Speer, N. (2021). Calculus Students' Deductive Reasoning and Strategies when Working with Abstract Propositions and Calculus Theorems. Primus, 31(2), 184–201. doi:10.1080/10511970.2019.1660931.

[19] Žilinskiene, I., & Demirbilek, M. (2015). Use of geogebra in primary math education in Lithuania: An exploratory study from teachers' perspective. Informatics in Education, 14(1), 127–142. doi:10.15388/infedu.2015.08.

[20] Bulut, M., Akçakın, H. Ü., Kaya, G., & Akçakın, V. (2016). The effects of geogebra on third grade primary students' academic achievement in fractions. Mathematics Education, 11(2), 327–335. doi:10.12973/iser.2016.2109a.

[21] Diković, L. (2009). Applications geogebra into teaching some topics of mathematics at the college level. Computer Science and Information Systems, 6(2), 191–203. doi:10.2298/CSIS0902191D.

[22] Kusumah, Y. S., Kustiawati, D., & Herman, T. (2020). The effect of GeoGebra in three-dimensional geometry learning on students' mathematical communication ability. International Journal of Instruction, 13(2), 895–908. doi:10.29333/iji.2020.13260a.

[23] Hohenwarter, M., Hohenwarter, J., Kreis, Y., & Lavicza, Z. (2008). Teaching and learning calculus with free dynamic mathematics software GeoGebra. 11th International Congress on Mathematical Education (ICME 11), 6-13 July, 2008, Monterrey, Mexico.

[24] Jelatu, S., Sariyasa, & Made Ardana, I. (2018). Effect of GeoGebra-aided REACT strategy on understanding of geometry concepts. International Journal of Instruction, 11(4), 325–336. doi:10.12973/iji.2018.11421a.

[25] Radović, S., Radojičić, M., Veljković, K., & Marić, M. (2020). Examining the effects of GeoGebra applets on mathematics learning using interactive mathematics textbook. Interactive Learning Environments, 28(1), 32–49. doi:10.1080/10494820.2018.1512001.

[26] Joshi, D. R., & Singh, K. B. (2020). Effect of using Geogebra on eight grade students' understanding in learning linear equations. Mathematics Teaching-Research Journal, 12(3), 76–83.

[27] Khalil, M., Khalil, U., & ul Haq, Z. (2019). Geogebra as a Scaffolding Tool for Exploring Analytic Geometry Structure and Developing Mathematical Thinking of Diverse Achievers. International Electronic Journal of Mathematics Education, 14(2), 427–434. doi:10.29333/iejme/5746.

[28] Farihah, U. (2015). The effect of the GeoGebra interactive program on students' motivation and learning outcomes on straight-line equation graph material. Jurnal Pendidikan Dan Pembelajaran Matematika, 1(1), 11–23. (In Indonesian).

[29] Khalil, M., Farooq, R. A., Çakiroglu, E., Khalil, U., & Khan, D. M. (2018). The development of mathematical achievement in analytic geometry of grade-12 students through GeoGebra activities. Eurasia Journal of Mathematics, Science and Technology Education, 14(4), 1453–1463. doi:10.29333/ejmste/83681.

[30] Alkhateeb, M. A., & Al-Duwairi, A. M. (2019). The Effect of Using Mobile Applications (GeoGebra and Sketchpad) on the Students' Achievement. International Electronic Journal of Mathematics Education, 14(3), 523–533. doi:10.29333/iejme/5754.

[31] Hendriana, H., Johanto, T., & Sumarmo, U. (2018). The role of problem-based learning to improve students' mathematical problem-solving ability and self-confidence. Journal on Mathematics Education, 9(2), 291–300. doi:10.22342/jme.9.2.5394.291-300.

[32] Tessmer, Martin, (1993). Planning and Conducting Formative Evaluations (1st Ed.). Routledge, London, United Kingdom. doi:10.4324/9780203061978

[33] Plomp, T., & Nievenn, N. (2013). Educational Design Research: An Introduction. Institute for Curriculum Development (SLO), Enschede, Netherlands.

[34] Arnawa, I. M., Yerizon, Y., & Nita, S. (2019). Improvement Students' Level of Proof Ability in Abstract Algebra Trough APOS Theory Approach. International Journal of Scientific and Technology Research, 8(7), 128–131.

[35] Fauziah, A., Putri, R. I. I., Zulkardi, & Somakim. (2020). Developing PMRI learning environment through lesson study for pre-service primary school teacher. Journal on Mathematics Education, 11(2), 193–208. doi:10.22342/jme.11.2.10914.193-208.

[36] Mahmud, R. (2017). The development of social learning model based on metacognitive strategies to foster mathematics self-efficacy of senior high school students 9 Makassar, Indonesia. Eurasia Journal of Mathematics, Science and Technology Education, 13(8), 4873–4883. doi:10.12973/eurasia.2017.00970a.

[37] Riduwan, M. B. A. (2007). Measuring Scale of Research Variables. Alf, Bandung, Indonesian. (In Indonesian)

[38] Murni, V., Sariyasa, S., & Ardana, I. M. (2017). GeoGebra Assist Discovery Learning Model for Problem Solving Ability and Attitude toward Mathematics. Journal of Physics: Conference Series, 895(1), 012049. doi:10.1088/1742-6596/895/1/012049.

[39] Dastjerdi, S., Akgöz, B., Civalek, Ö., Malikan, M., & Eremeyev, V. A. (2020). On the non-linear dynamics of torus-shaped and cylindrical shell structures. International Journal of Engineering Science, 156, 103371. doi:10.1016/j.ijengsci.2020.103371.

[40] Chen, C. W. K., Chen, C., & Shieh, C. J. (2020). A study on correlations between computer-aided instructions integrated environmental education and students' learning outcome and environmental literacy. Eurasia Journal of Mathematics, Science and Technology Education, 16(6), em1858. doi:10.29333/EJMSTE/8229.

[41] Haciomeroglu, E. S., & Chicken, E. (2012). Visual thinking and gender differences in high school calculus. International Journal of Mathematical Education in Science and Technology, 43(3), 303-313. doi:10.1080/0020739X.2011.618550.

[42] Serin, O. (2011). The effects of the computer-based instruction on the achievement and problem solving skills of the science and technology students. Turkish Online Journal of Educational Technology, 10(1), 183–201.

[43] Hidayati, D. W., & Kurniati, L. (2018). The Influence of Self-Regulated Learning to Mathematics Critical Thinking Ability on 3D-Shapes Geometry Learning using Geogebra. JIPM (Jurnal Ilmiah Pendidikan Matematika), 7(1), 40. doi:10.25273/jipm.v7i1.2965.

[44] Dwijayani, N. M. (2020). Meningkatkan Kemampuan Pemecahan Masalah Mahasiswa dengan Menggunakan Geogebra. Jurnal Pendidikan Matematika, 11(1), 110. doi:10.36709/jpm.v11i1.10045. (In Indonesian).

[45] Supriadi, N. (2015). Pembelajaran geometri berbasis geogebra sebagai upaya meningkatkan kemampuan komunikasi matematis siswa madrasah tsanawiyah (MTs). *Al-Jabar: Jurnal Pendidikan Matematika*, *6*(2), 99-110.. doi:10.24042/ajpm.v6i2.20. (In Indonesian).

[46] del Cerro Velázquez, F., & Morales Méndez, G. (2021). Application in augmented reality for learning mathematical functions: A *study* for the development of spatial intelligence in secondary education students. Mathematics, 9(4), 369. doi:10.3390/math9040369.

[47] Buckley, J., Seery, N., & Canty, D. (2019). Investigating the use of spatial reasoning strategies in geometric problem solving. International Journal of Technology and Design Education, 29(2), 341–362. doi:10.1007/s10798-018-9446-3.

[48] Juandi, D., Kusumah, Y. S., Tamur, M., Perbowo, K. S., & Wijaya, T. T. (2021). A meta-analysis of GeoGebra software decade of assisted mathematics learning: what to learn and where to go?. Heliyon, 7(5), e06953. doi:10.1016/j.heliyon.2021.e06953.

[49] Arnis, F. M., Syahputra, E., & Surya, E. (2019). Analysis of Trajectory Thinking of Middle School Students to Complete the Problem of Spatial Ability with Realistic Mathematical Education Learning. Journal of Education and Practice, 10(20), 103–109. doi:10.7176/jep/10-20-12.